The Fundamental Course

Chapter:

Weekly Quiz 2 Solutions
(Proof Techniques)

- By Deepak Poonia(IISc Bangalore)

Instructor:

Deepak Poonia

IISc Bangalore

GATE CSE AIR 53; AIR 67; AIR 206; AIR 256;

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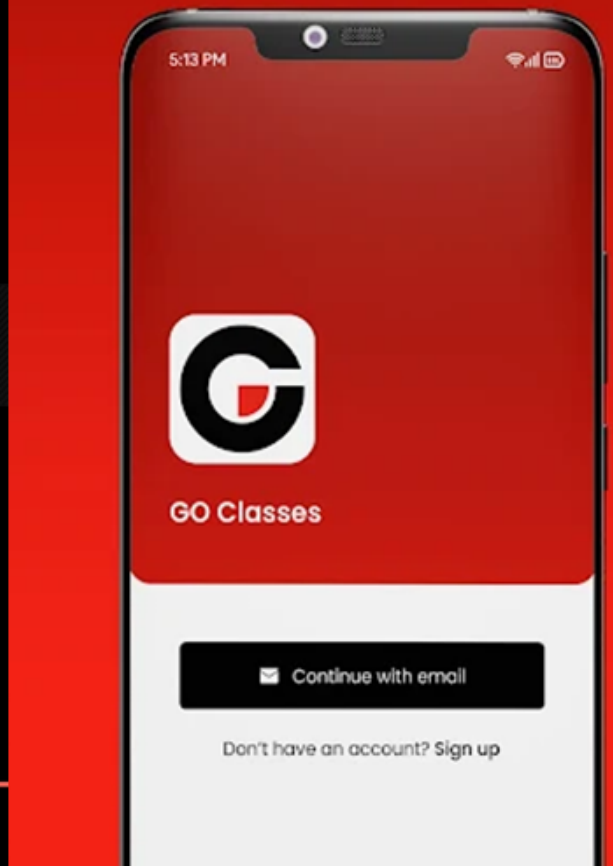

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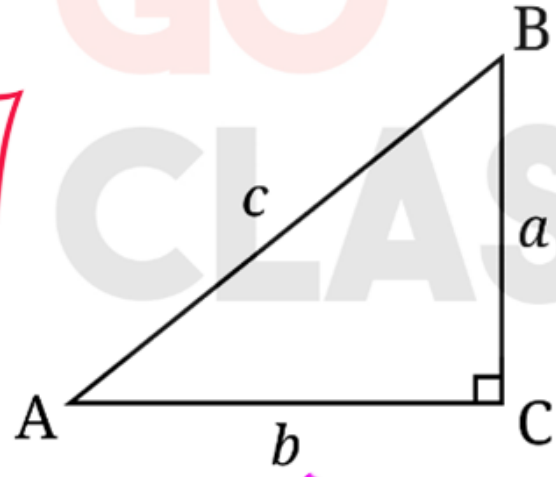
5

Consider the following right-angled triangle, in which the angle between the side AC and CB is a right angle. Let a, b, c be the length of the sides BC, AC, AB respectively. Assume a, b, c are integers.



Note that

$abc = a \times b \times c$

a OR b is even

Which of the following is/are correct statements?

A. (abc) is even. ✓

B. (bc) is even. ✗

C. (ab) or (ac) is even.

D. Both a, b are even. ✗

$a = 4, b = 3, c = 5$

even

WQ 2 Q7.

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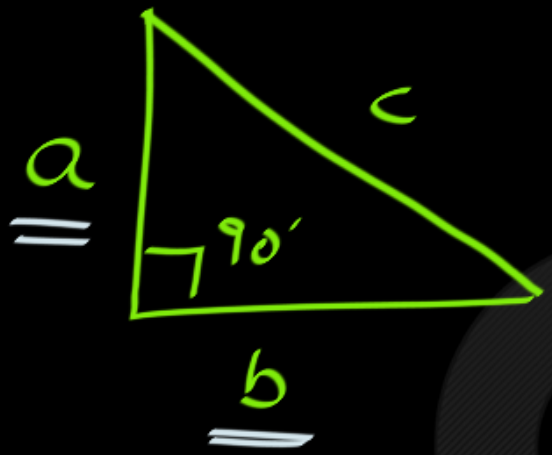
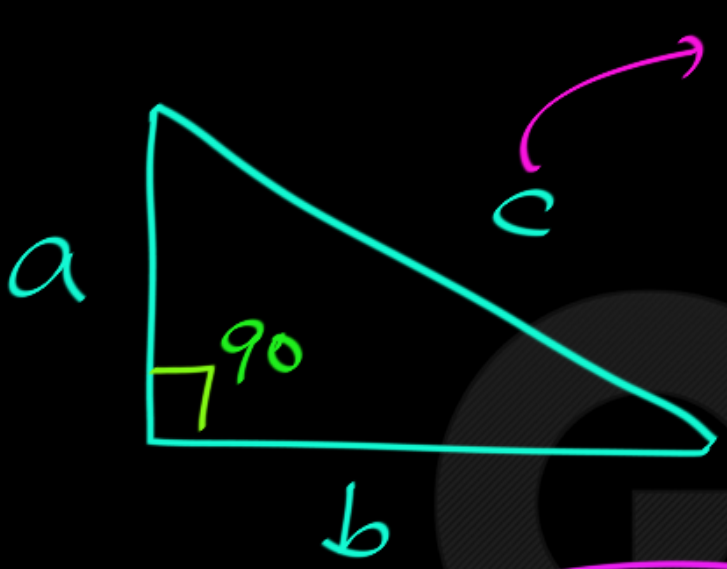
a, b, c are Integers.



HW 2 : In any Right Angle Triangle, if all sides are integers then at least one of the two shorter sides is even.



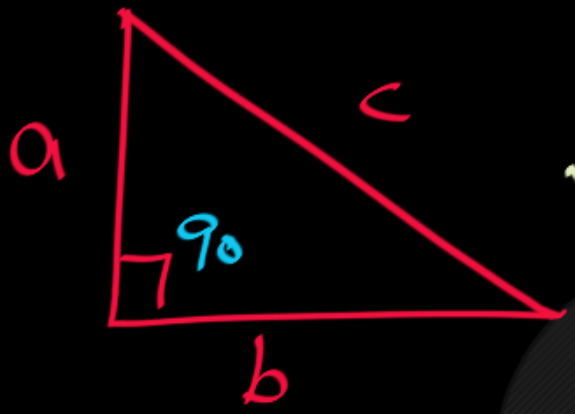
$$a^2 + b^2 = c^2$$

If $a, b, c \in \mathbb{Z}$ then at least one of a, b is even.

$$(\text{odd})^2 + (\text{odd})^2 \neq (\text{int})^2$$

Any int

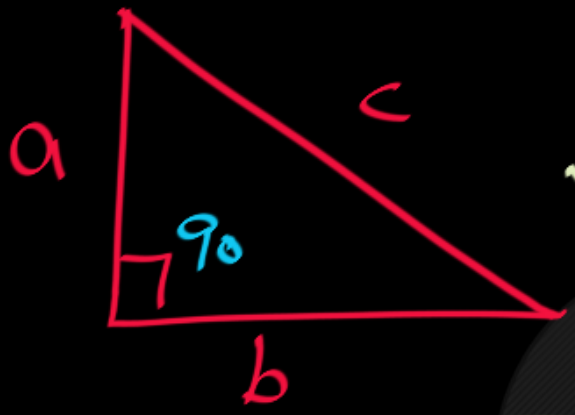
Summation of two odd squares is Never a perfect square.



$a, b, c \in Z$

~~①~~ a or b is even.

$a, b, c \in Z$



(1) a or b is even.

Ep1: $a=3, b=4, c=5$ → odd

Ep2: $a=6, b=8, c=10$ → even

(2) a, b both even ? NoT Necessary

Q: Which is Correct ? $a, b \in Z$

(1) $a + b = b + a$

(2) $2a = a^2$

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Q: Which is Correct ?

$a, b \in Z$

① $a + b = b + a$ ✓

② $2a = a^2$ ✗

$a = 2$ ✓

$a = 3$ ✗

may or may not be

Note :

If you say Stetement S is True or Correct then S must be Always Correct.

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3

Let n be a positive integer.

Which of the following statements is/are true?

A. If m, a are positive integers and $m \mid a^n$, then $m^n \mid a^n$.
B. If p is a prime and a is a positive integer and $p \mid a^n$, then $p^n \mid a^n$.
C. If t is odd, then t^n is also odd, for all n.
D. If t is even, then t^n is also even, for all n.

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D. If t is even, then t^n is also even, for all n.

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Ans: B, C, D

prove by Induction

A $n \in Z^+ \rightarrow n \in \{1, 2, 3, \ldots\}$

$m, a \in Z^+ \rightarrow$ Positive integers $= 1, 2, 3, 4, \ldots$

If $m \mid a^n$ then $m^n \mid a^n$ → false

↳ Disproof by Counter example :

$m = 4 ; \; a = 2 ; \; n = 5$

$4 \mid 2^5$ ~ But $4^5 \nmid 2^5$

(B) p : prime ; $a, n \in Z^+$

If $p \mid a^n$ then $p^n \mid a^n$.

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Euclid's lemma:

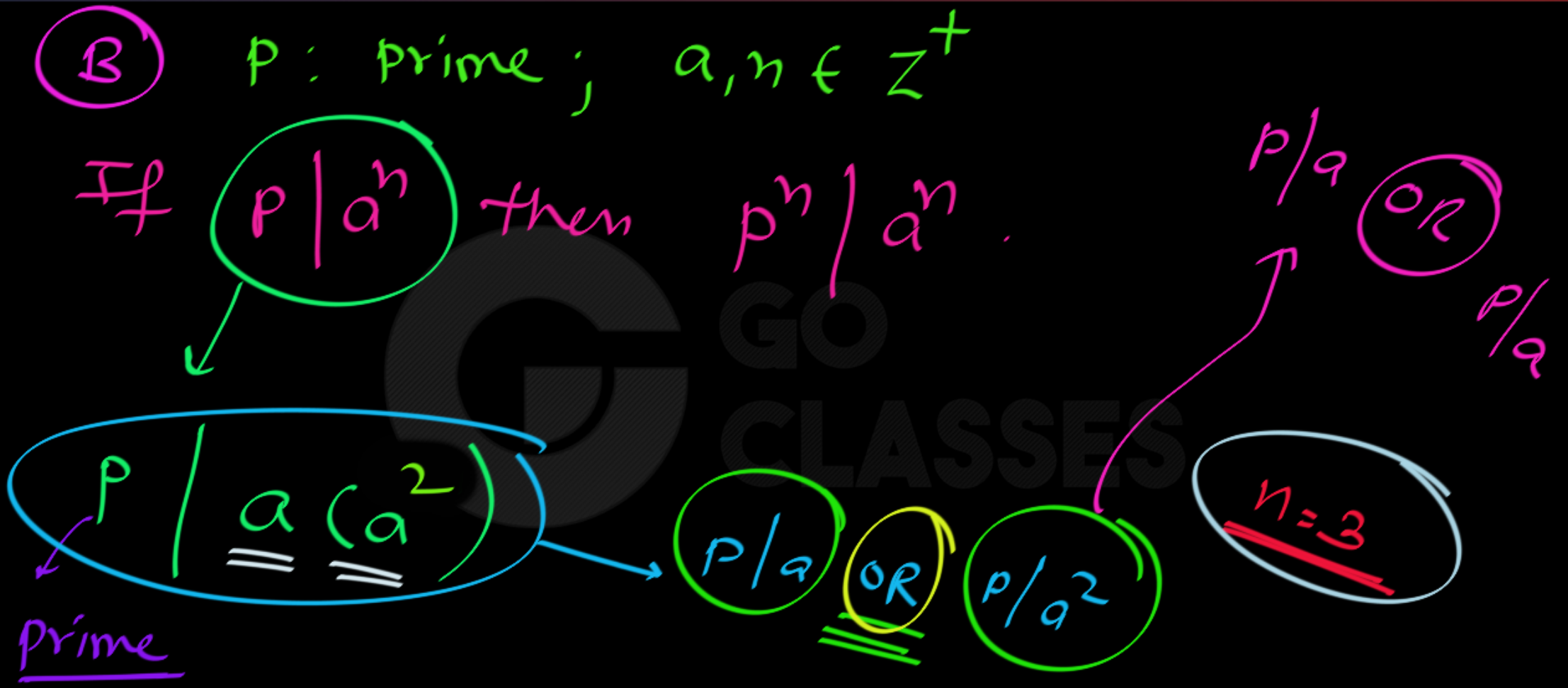
p: prime

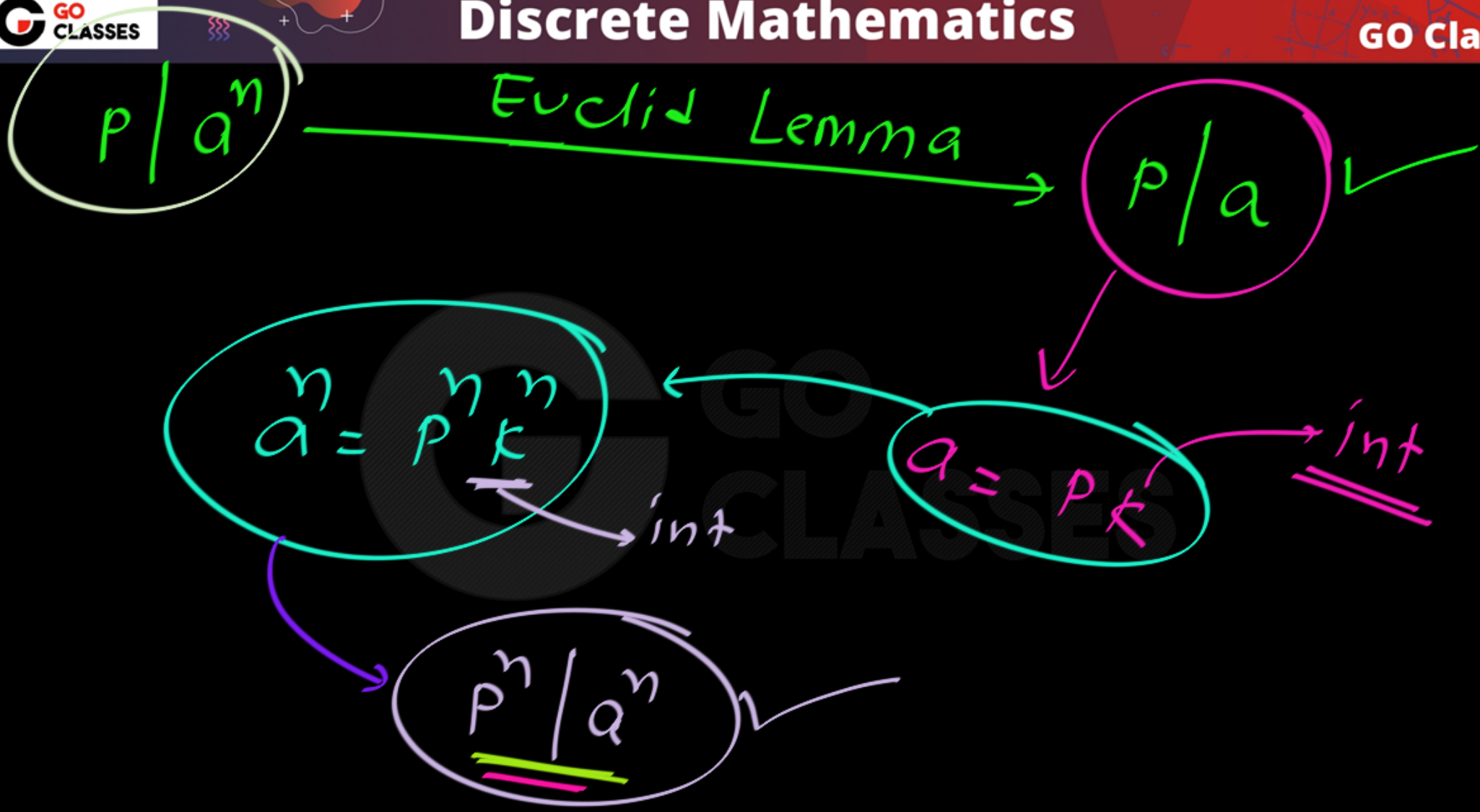
If $p \mid ab$ then $p \mid a$ OR $p \mid b$. True

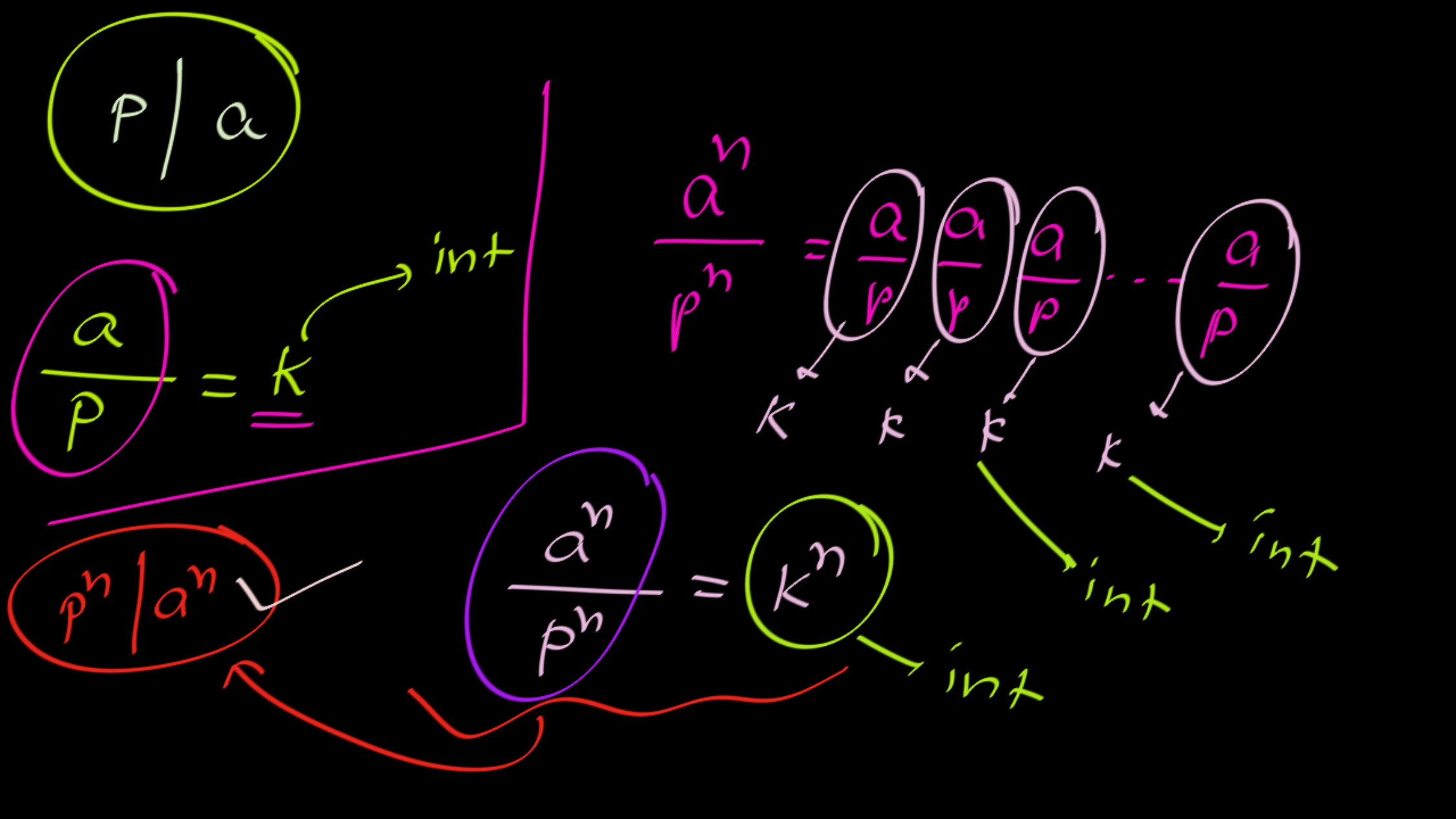
---

m : integer

If $m \mid ab$ then $m \mid a$ OR $m \mid b$ False


B
p : prime ; a, n ∈ Z+
If p | a^n then p^n | a^n .
p | a (a^2)
Prime
p | a
OR
p | a^2
p | a
OR
p | a
n = 3


$p \mid a^n$
Euclid Lemma
$p \mid a$
$a^n = p^n k^n$
int
$a = pk$
int
$p^n \mid a^n$


p | a
a/p = k
int
a^n/p^n = a/p · a/p · a/p · · · a/p
k k k k
int int int
a^n/p^n = k^n
int
p^n | a^n

(3) t : odd ; $n \in Z^+ = \{1, 2, 3, 4, \ldots\}$

$t^n = ?$

$n = 4$; $t^4 = t\,t\,t\,t = (\text{odd})(\text{odd})(\text{odd})(\text{odd})$

odd

odd · odd = odd

odd

odd

$(\text{odd})^{n}$ $\longrightarrow$ positive int

$= \text{odd}$

prove by Induction

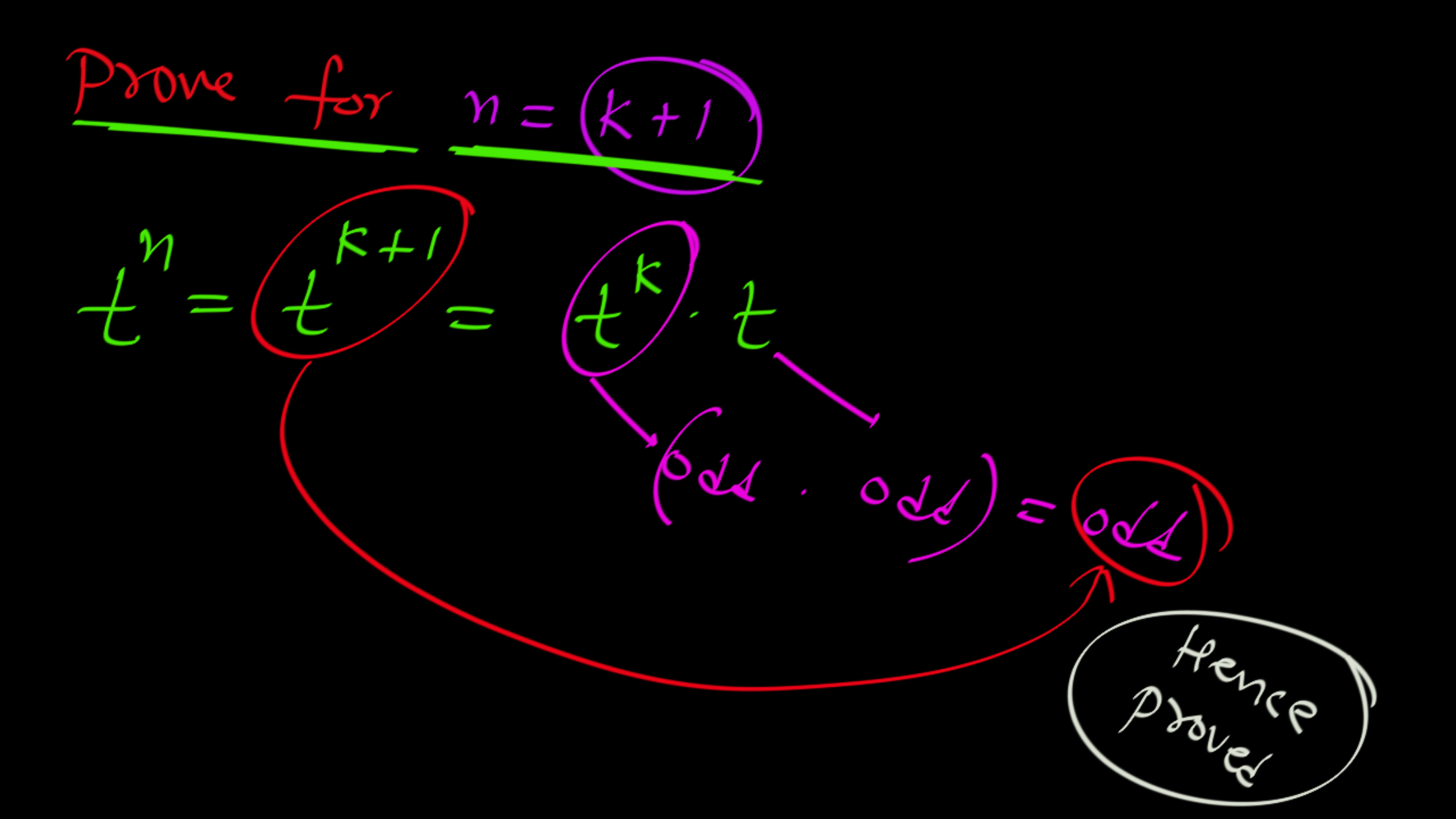
odd · odd = odd ✓

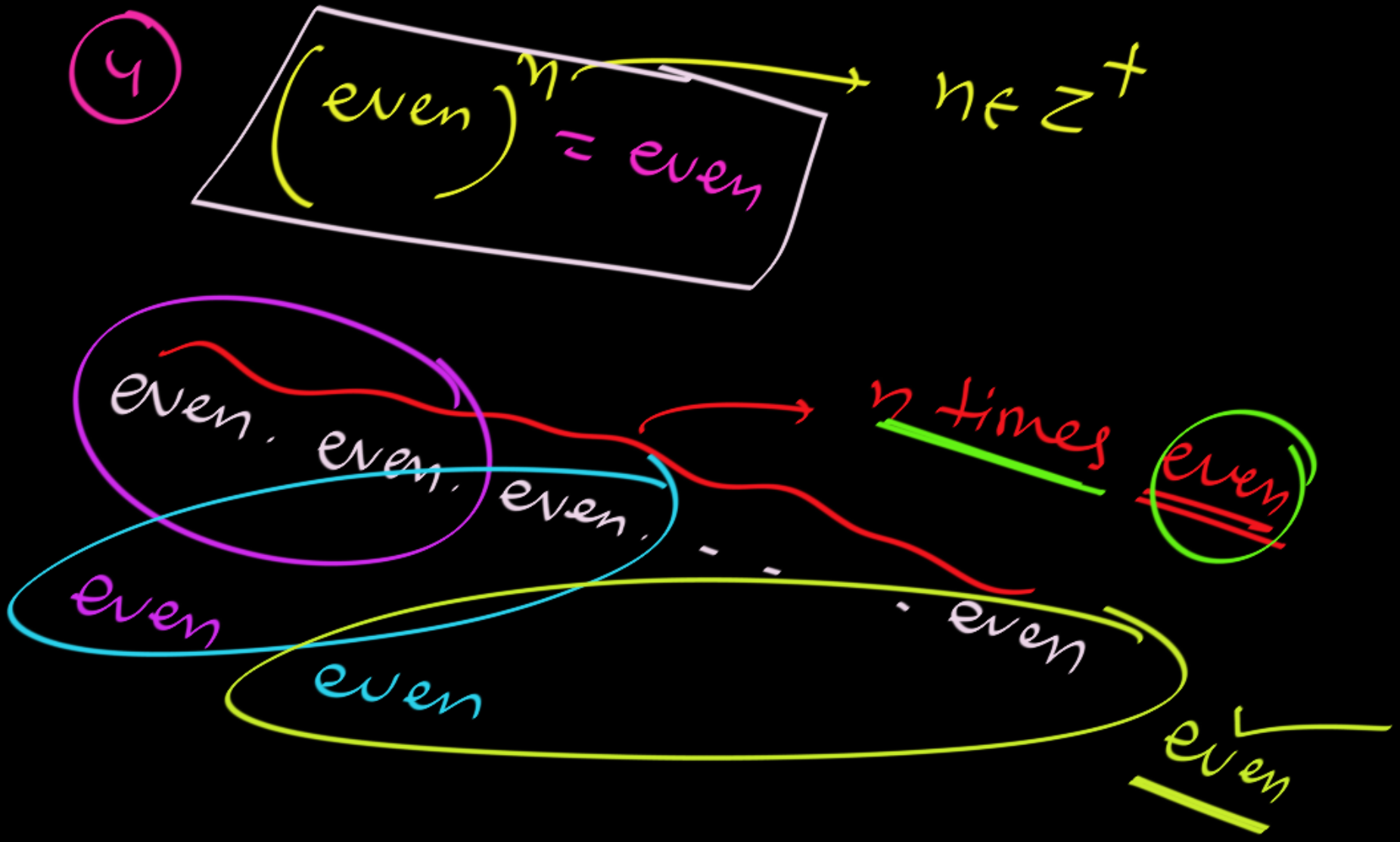
$n \in Z^+$

t : odd then t^n = odd → prove by Induction.

Base Case: $n = 1$; $t^1 = (odd)^1 = odd$ ✓

Assume that for $n \leq k$; $t^k = odd$


Prove for n = k+1
$t^n = t^{k+1} = t^k \cdot t$
(odd · odd) = odd
Hence proved


4
(even)^n = even
n ∈ Z+
even . even . even . . . . even
n times
even
even
even
even

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24 views

1

Consider the following statements:

1. If $n \in \mathbb{Z}$, then $5n^2 + 3n + 7$ is odd.
2. If $n \in \mathbb{Z}$, then $n^2 + 3n + 4$ is even.

HW

Which of the above statements is true?
(Hint: n can be either even or odd)

A. Only 1
B. Only 2
C. Both 1 & 2
D. None

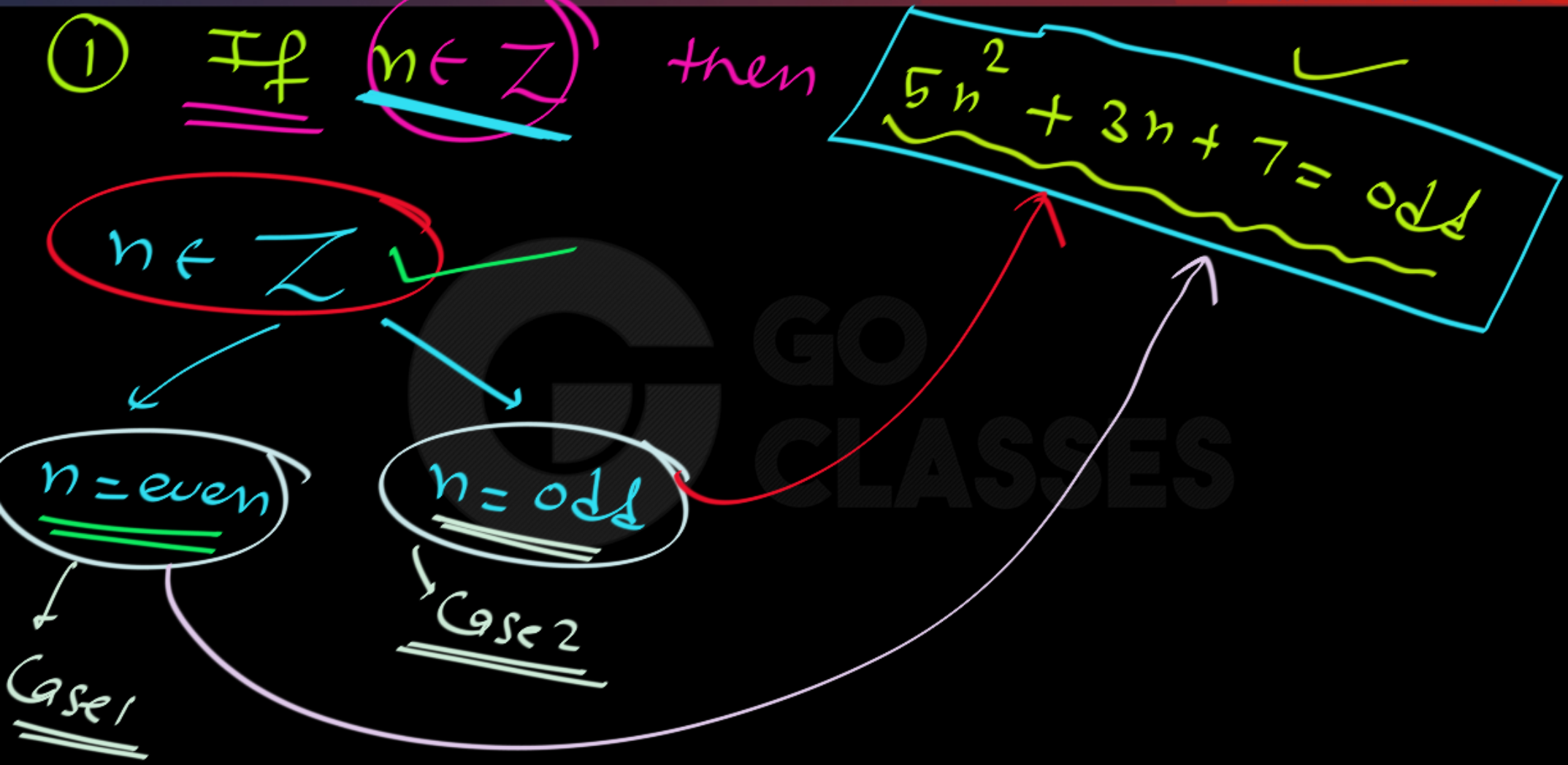
(1) If $n \in Z$ then $5n^2 + 3n + 7 = \text{odd}$

for Contradiction: for Some integer $n = k$

$5k^2 + 3k + 7 = \text{even}$

$5k^2 + 3k + 7 = 2a$

Stuck!!


1
If
n ∈ Z
then
5n² + 3n + 7 = odd
n ∈ Z
n = even
n = odd
Case 1
Case 2

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5

Consider the following statements:

- S1 : Given any $x \in \mathbb{R}$, there exists an element $y \in \mathbb{R}$ for which $xy = 1$.
- S2 : There exists two irrational numbers x and y such that $x + y$ is rational.

Which of the above statements is true?

A. Only S1
B. Only S2
C. Both S1 & S2
D. None

GO Classes Weekly Quiz 2 | Fundamental Course | Question: 5

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5

Consider the following statements:

- S1 : Given any $x \in \mathbb{R}$, there exists an element $y \in \mathbb{R}$ for which $xy = 1$.
- S2 : There exists two irrational numbers x and y such that $x + y$ is rational.

Which of the above statements is true?

A. Only S1
B. Only S2
C. Both S1 & S2
D. None

Real Number

false

GATE 2001 Question

S1

for all $x \in R$, there exist $y \in R$ such that $xy = 1$.

$x = 2 \rightsquigarrow y = \frac{1}{2}$

$x = -3.2 \rightsquigarrow y = -\frac{1}{3.2}$

$x \rightsquigarrow y = \frac{1}{x}$ → problem?

$x = 0$ → $xy = 1$ → No

$x = 0$ → Any y??

$x = 0$?

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S2

there Exist irrational a, b such that a + b = Rational.

$a = \sqrt{2}$

$b = -\sqrt{2}$

$a = 10 + \sqrt{2}$

$b = 10 - \sqrt{2}$

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2

Two integers have the same parity if they are both even or they are both odd. Otherwise, they have opposite parity. Which of the following statements is/are true?

A. If two integers have the same parity, their sum is odd.

B. If two integers have opposite parity, then their sum is odd.

C. If n is an odd integer, then $8 \mid n^2 - 1$.

D. Suppose $a, b \in \mathbb{Z}$. If ab is odd, then $a^2 + b^2$ is even.

odd + even = odd

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Integers a, b

Same Parity : both odd OR both even.

Opposite Parity : one even, one odd.

odd + odd = even
even + even = even

} ⊢

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(c) If n = odd then $8 \mid n^2 - 1$

Direct proof:

$n = 2k + 1$

$n^2 - 1 = 4k^2 + 4k = 4k(k+1)$

k and $k+1$ are consecutive

even . int = even

$4k(k+1) = 4 \times 2m$

$4 \times 2m$ → Divisible by 8

even = $2m$, int

(D) $a, b \in Z$

If $ab = odd$ ⟹ both a, b odd

then $a^2 + b^2 = ?$

odd + odd = even

$(odd)^2 = odd$

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68 views

A, B, C, D

2

Which of the following statements is/are True?

A. If x and y are two integers whose product is odd, then both must be odd.

B. If a and b are real numbers such that the product ab is an irrational number, then either a or b must be an irrational number.

C. For any integer m, if m^2 is even, then m is even.

D. The sum of a rational number and an irrational number is irrational.

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Q1:

Rational + Irrational ≠ Rational

by Contradiction → Assume:

Rational + Irr. = Rat.

$\frac{a}{b}$ + Irr = $\frac{c}{d}$

$b \neq 0$ ← $\frac{a}{b}$ → $d \neq 0$

$$\frac{a}{b} + \text{Irr} = \frac{c}{d}$$

$$\text{Irr} = \frac{c}{d} - \frac{a}{b} = \frac{cb - ad}{bd}$$

$cb - ad$: int

bd : int, $\neq 0$

Contradiction :

$$\text{Irr} = \frac{\text{int}}{\text{int}}$$

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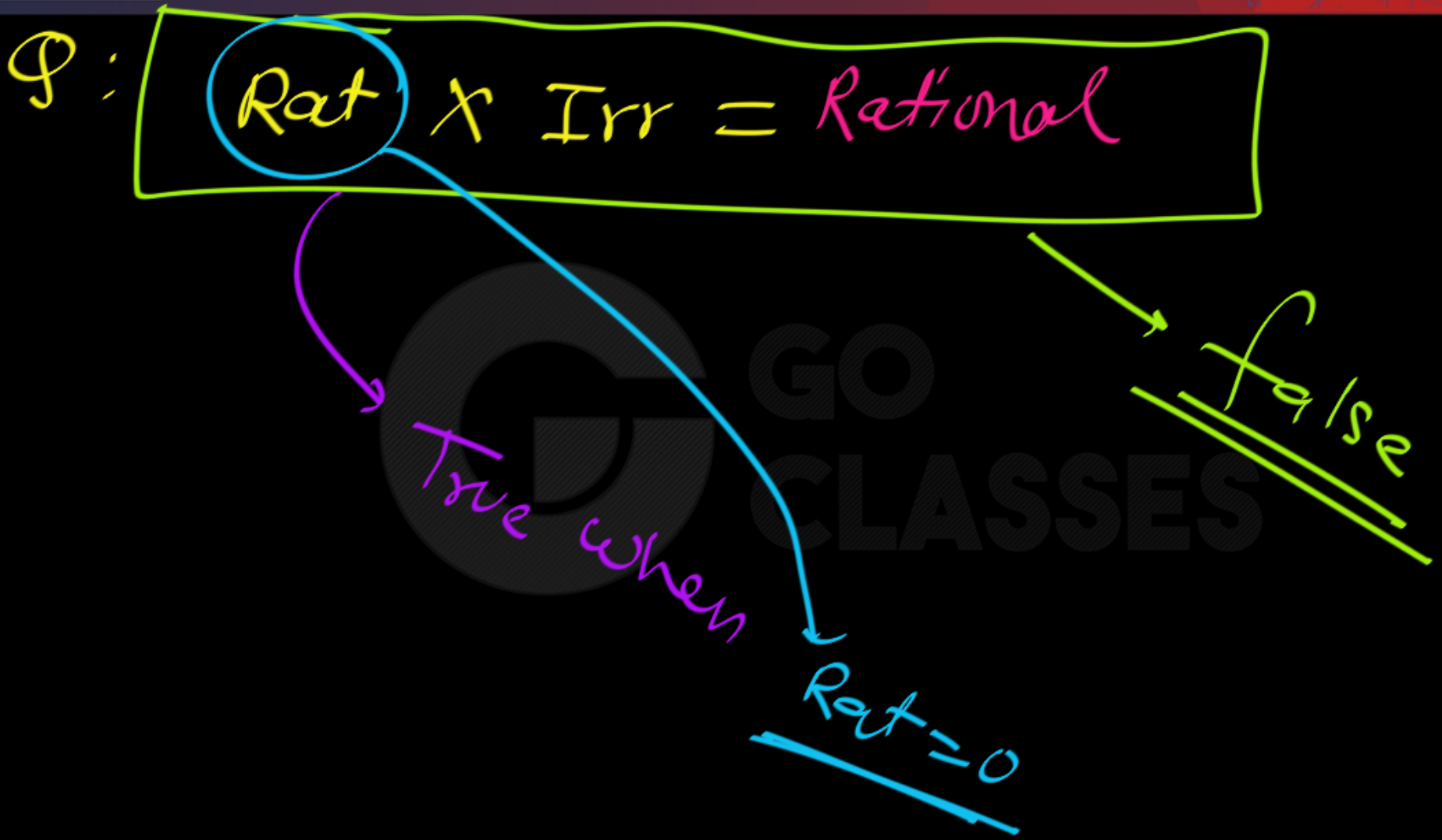
Q: Rat × Irr = Rational

Is it Possible? Yes

Take Rational = 0


Q :
Rat × Irr = Rational
False
True when
Rat = 0

Q : Rat × Irr = Rational

Non-Zero

Never Happen

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Note : $a \longrightarrow$ Non Zero, Rational

$b \longrightarrow$ Irrational

a^b = Irrational

Prove by Contradiction

$\frac{m}{n}$ Irr $= \frac{p}{q}$

Contradiction

Irrational + Irrational = may or may not be Rational

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(b)

Rational · Rational = Rational

If a^b = Irrational

then a or b must be Irrational.

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2

Consider the following statements:

- S1 : Suppose $a, b, c \in \mathbb{Z}$. If $a^2 \mid b$ and $b^3 \mid c$, then $a^6 \mid c$.
- S2 : Suppose $a \in \mathbb{Z}$. If a^3 is not divisible by 8, then a is odd.

Which of the above statements is true?

A. Only S1.
B. Only S2.
C. Both S1 & S2.
D. None

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2

Consider the following statements:

- S1 : If $a, b \in Z$ and $a + b$ is even, then $a^2 + b^2$ is even.
- S2 : If $a, b \in Z$ and $a + b$ is odd, then $a^2 + b^2$ is odd.

Which of the above statements is true?

A. Only S1
B. Only S2
C. Both S1 & S2
D. None

GO Classes Weekly Quiz 2 | Fundamental Course | Question: 3

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1

Consider the following statements:

- S1 : Suppose $a, b \in \mathbb{Z}$. If $\left(a^2\right)\left(b^2-2b\right)$ is odd, then both a and b are odd.
- S2 : Suppose $a, b \in \mathbb{Z}$. If $25 \nmid ab$, then $5 \nmid a$ or $5 \nmid b$.

Which of the above statements is true?

A. Only S1
B. Only S2
C. Both S1 & S2
D. None

S1: $a, b \in Z$

If $(a^2)(b^2 - 2b) = \text{odd}$ then both a, b odd.

even * int = even

Contraposition: a or b is even.

a = even ✓

$a \cdot a(b^2 - 2b)$

even

b = even ✓

$a^2 b(b-2)$ — even

even

GO Classes Weekly Quiz 2 | Fundamental Course | Question: 4

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1

Consider the following statements:

- S1 : If an integer ' a ' is even then $\left(a^2\right)+2a+9$ is odd.
- S2 : For an integer ' a ', if $\left(a^2\right)+2a+9$ is odd then ' a ' is even.

Which of the above statements is true?

A. Only S1
B. Only S2
C. Both S1 & S2
D. None

Extra:

True/False??

Let a,b be two integers. If a|b & b|a then a=b.

Extra:

True/False??

Let a,b be two integers. If a|b & b|a then a=b.

false

$a = 2$
$b = -2$

$a|b$
$b|a$

But $a \neq b$

Extra:

True/False??

Let a,b be two integers. If a|b & b|a then a=b.

positive

True

How to question

Extra: → an interesting result

Prove every odd integer is the difference of two squares.

Proof:

$(2k+1) = (k+1)^2 - k^2$

Perfect square ≡ square

$(2k+1) = (k+1)^2 - k^2$

odd int

⇒ This type of proof is called Constructive proof.

Hope you liked the Weekly Quiz.

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